

Applied Math Dictionary

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Based on lectures by Nir Gavish

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These notes are not endorsed by the lecturers. I have revised them outside lectures to incorporate supplementary explanations, clarifications, and material for fun. While I have strived for accuracy, any errors or misinterpretations are most likely mine.

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Disclaimer: The course “Applied Mathematics” is one of the few courses that does not try to formalize the math behind its methods. In such courses, I believe that the best method to succeed is to learn the language of the course, and practice exercises in order to get a feeling for the methods. Indeed, it is not the methods in this course which are not fully formalized, it’s mostly the language. As such, I suggest to treat the terms in this course as dictionary entries rather than mathematical definitions. Of course, formal definitions and theorems will be introduced when deemed fit.

1 Dimensional analysis

Entry 1.1 (Dimension). In this course, dimension refers to “physical units” such as $[L]$ (Length), $[T]$ (time) etc. For example, the radius of the earth is denoted by $R[L]$, where R is a value and L is the dimension of length. Dimensions are denoted by capital letters.

Entry 1.2 (Relatively very small). We say that x is very small relative to R if it is intuitively a lot smaller, and we will not formalize this notion further in this course. In such a case, we denote $x \ll R$. Sometimes we need to use “common sense”, for example, if R_1 is the radius of the earth, and R_2 is the distance from the earth to the sun, then $R_1 \ll R_2$.

Entry 1.3 ((non)dimensional equation). A dimensional equation is an equation which considers dimensions. For instance, the equation for distance:

$$x[L] = v[LT^{-1}] \times t[T],$$

is a dimensional equation. A nondimensional equation is an equation where we do not consider dimensions such as Euler’s formula:

$$e^{\pi i} + 1 = 0.$$

Entry 1.4 (Regular perturbation). A regular perturbation is a small variable (possibly constant) that has a negligible effect on the solution. For example if $100 \ll R$ is a variable, and the actual solution is $S(R) = R + 1$ then we could say 1 is a regular perturbation.

Entry 1.5 (Singular perturbation). A singular perturbation is a small variable in the equation which affects the solution drastically. For example, if our problem is finding the number of zeros of the equation:

$$\epsilon x^2 - 2x + 1 = 0$$

then for small nonzero values of ϵ the result is 2, but when $\epsilon = 0$ the solution changes drastically to 1. Thus ϵ is a singular perturbation.

Entry 1.6 (Dimensionally complete equation). A dimensionally complete equation is an equation where the RHS and the LHS both have the same dimension such as:

$$x[L] = v[LT^{-1}] \times t[T].$$

Equations of the form:

$$x[L] = v[LT^{-1}] \quad \text{or} \quad u(x, t) = x[L] + t[T],$$

are called dimensionally incomplete.

Entry 1.7 (Nondimensionalization). Nondimensionalization is the process of changing a dimensional equation into a nondimensional equation. This is done by normalizing the dimensional variable by some parameter. For example if $x[L]$ is the height of a rocket, and $H[L]$ is its maximal height, then $\tilde{x} = \frac{x}{H}$ is dimensionless. By plugging $x = \tilde{x}H$ into the dimensional equation, and considering the equation in $\tilde{x}$ we would get a nondimensional equation. As you can see, this process is not deterministic since different normalizations (also called scalings) give different equations.

Entry 1.8 (The $\mathcal{O}(1)$ criterion). The $\mathcal{O}(1)$ criterion states that in the process of nondimensionalization, in order to reach a solution that “makes sense”, we need to have dimensionless variables that are $\mathcal{O}(1)$. In this case, a variable x is called $\mathcal{O}(1)$ if $-1 \leq x \leq 1$ (we do not use the standard definition of $\mathcal{O}(1)$ for obvious reasons).

Entry 1.9 (Unit of measurement). A unit of measurements is a certain arbitrary size that allows us to assess dimensions. For example, meters, centimeters, my current height, are all equally valid units of measurement for the dimension of length.

Entry 1.10 (Base unit).

Entry 1.11.